Forces and Equilibrium

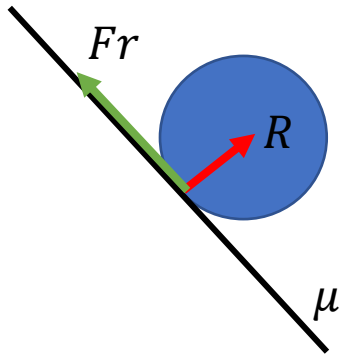
KEY POINTS – Must Remember

Take your time with the free body diagrams

$\nearrow \sum F = 0$

$\curvearrowright \sum M = 0$

Static Friction

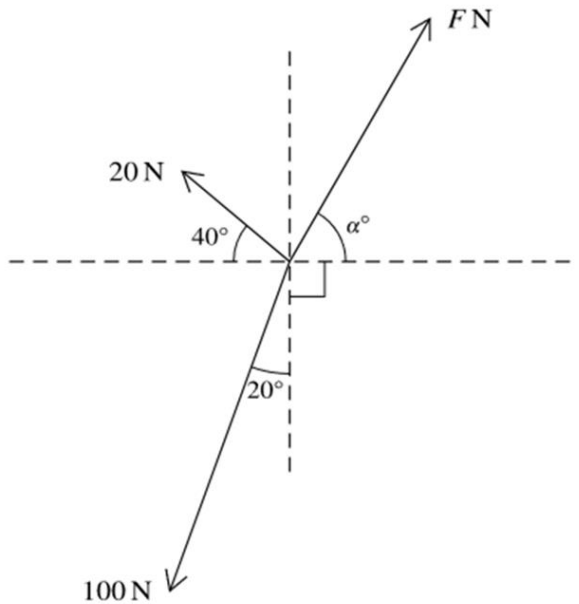


$Fr \leq \mu R$

$Fr_{max} = \mu R$

Forces and Equilibrium – Key Question

High School Level



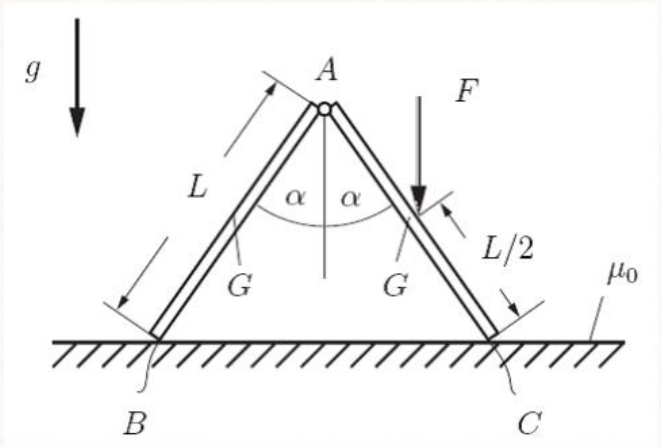
1 Three coplanar forces of magnitudes 20 N, 100 N and F N act at a point. The directions of these forces are shown in the diagram.

Given that the three forces are in equilibrium, find F and α.

Forces and Equilibrium – Key Question

University Level

Two homogeneous, slim bars (negligible thickness) of weight G each are hinged together without friction at their upper ends at point A. Additionally, a load F is applied to the right bar in vertical direction. The system rests under the influence of gravity on a horizontal plane. At the contact points B and C friction is to be considered (sticking friction coefficient μ_0).



For the displayed system, the condition for the angle α is to be determined such that the system remains in static equilibrium.

(a) Draw suitable free body diagrams for the system.
(b) State the equations for static equilibrium.
(c) Resolve the equations for the normal forces and friction forces at the contact points B and C.
(d) Use Coulomb's law to obtain a condition for α such that equilibrium is possible.

Kinematics

KEY POINTS – Must Remember

Determine if the problem deals with constant acceleration or non-constant acceleration.

Constant Acceleration	Non-Constant Acceleration
$v = u + at$	$v = \frac{ds}{dt}$
$s = ut + \frac{1}{2}at^2$	$a = \frac{dv}{dt}$
$v^2 = u^2 + 2as$	$a = v\frac{dv}{ds}$
$s = \frac{1}{2}(u + v)t$	

Kinematics – Key Question

High School Level

1 A car starts from rest and moves in a straight line with constant acceleration for a distance of 200 m, reaching a speed of 25 m s^{-1}. The car then travels at this speed for 400 m, before decelerating uniformly to rest over a period of 5 s.

(a) Find the time for which the car is accelerating.

(b) Sketch the velocity–time graph for the motion of the car, showing the key points.

(c) Find the average speed of the car during its motion.

2 A particle starts from a point O and moves in a straight line. The velocity v m s^{-1} of the particle at time t s after leaving O is given by

$$v = k(3t^2 - 2t^3),$$

where k is a constant.

(a) Verify that the particle returns to O when $t = 2$.

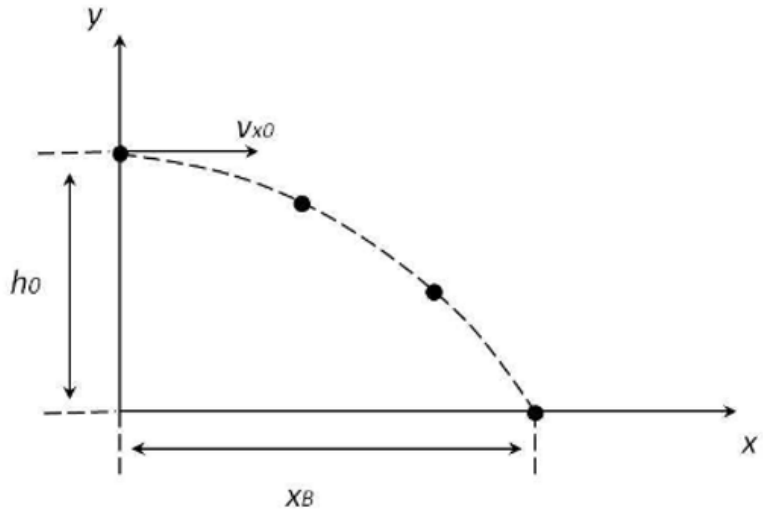
(b) It is given that the acceleration of the particle is -13.5 m s^{-2} for the positive value of t at which $v = 0$.

Find k and hence find the total distance travelled in the first two seconds of motion.

Kinematics – Key Question

University Level

A mass m moves horizontally at the time $t_0 = 0$. Its initial height above ground is $h_0 = \text{const}$ and its initial horizontal velocity is $v_{x0} = \text{const}$. Furthermore the vertical velocity v_y is equal to zero at the beginning (see sketch).
Obviously the mass looses height due to gravitation. With respect to air resistance the motion of the mass can be described by the formulas



$$ma_x = -mc_0 v_x^2 \qquad (1)$$

and

$$ma_y = -mg\ . \qquad (2)$$

Equation (1) describes the horizontal and equation (2) the vertical motion. Furthermore c_0 and g are constants.

a) Compute the time t_B at which the mass touches the ground in dependency of c_0 and v_{x0}.
b) Compute the distance x_B which describes the position where the mass touches the ground.

Momentum

KEY POINTS – Must Remember

Take your time with the free body diagrams
↗ $P = mv$ $Total\ Initial\ Momentum\ =\ Total\ final\ momentum$

Momentum – Key Question

High School Level

1 Two particles A and B, of masses 0.4 kg and 0.2 kg respectively, are moving down the same line of greatest slope of a smooth plane. The plane is inclined at 30° to the horizontal, and A is higher up the plane than B. When the particles collide, the speeds of A and B are $3\,\text{m s}^{-1}$ and $2\,\text{m s}^{-1}$ respectively. In the collision between the particles, the speed of A is reduced to $2.5\,\text{m s}^{-1}$.

(a) Find the speed of B immediately after the collision.

After the collision, when B has moved 1.6 m down the plane from the point of collision, it hits a barrier and returns back up the same line of greatest slope. B hits the barrier 0.4 s after the collision, and when it hits the barrier, its speed is reduced by 90%. The two particles collide again 0.44 s after their previous collision, and they then coalesce on impact.

(b) Show that the speed of B immediately after it hits the barrier is $0.5\,\text{m s}^{-1}$. Hence find the speed of the combined particle immediately after the second collision between A and B.

Momentum – Key Question

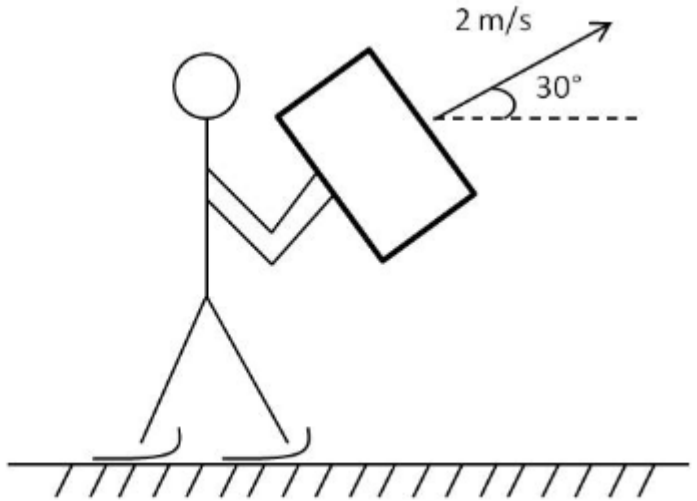
University Level

A man with mass of 70 kg wearing ice skates throws an 8 kg block with an initial velocity of 2 m/s, measured relative to himself, in the direction shown in the figure. If he is originally at rest and completes the throw in 1.5 s while keeping his legs rigid,

a) determine the horizontal velocity of the man just after releasing the block.

b) Determine the vertical reaction force of both his skates on the ground during the throw.

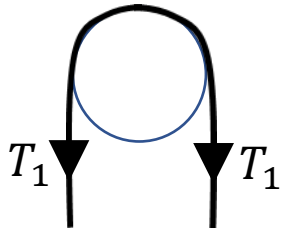
Neglect the friction and the motion of his arms.



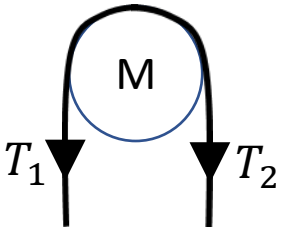
Newton‘s Laws

KEY POINTS – Must Remember

Take your time with the free body diagrams



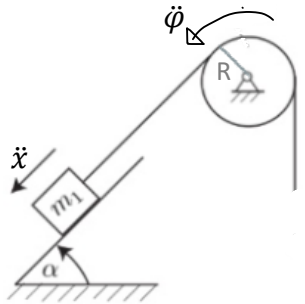
Massless pulley : Tension same on both sides



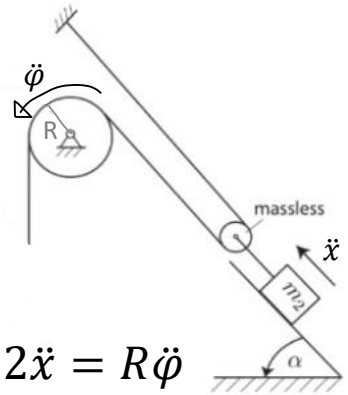
Pulley with mass: Tension different on both sides

$\nearrow \sum F = ma$

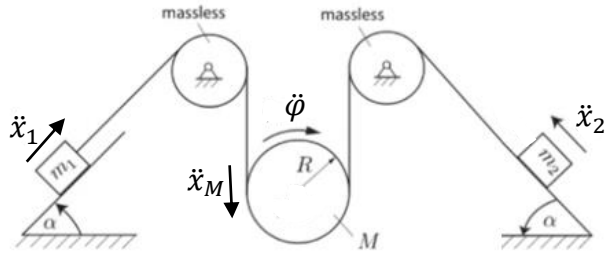
$\curvearrowright \sum M = J\alpha$



$\ddot{x} = R\ddot{\varphi}$



$2\ddot{x} = R\ddot{\varphi}$



$\ddot{x}_M = \frac{1}{2}(\ddot{x}_1 + \ddot{x}_2)$

$\ddot{x}_2 - \ddot{x}_1 = R\ddot{\varphi}$

Newton‘s Laws – Key Question

High School Level

1 Two particles P and Q, of masses 0.5 kg and 0.3 kg respectively, are connected by a light inextensible string. The string is taut and P is vertically above Q. A force of magnitude 10 N is applied to P vertically upwards.

Find the acceleration of the particles and the tension in the string connecting them.

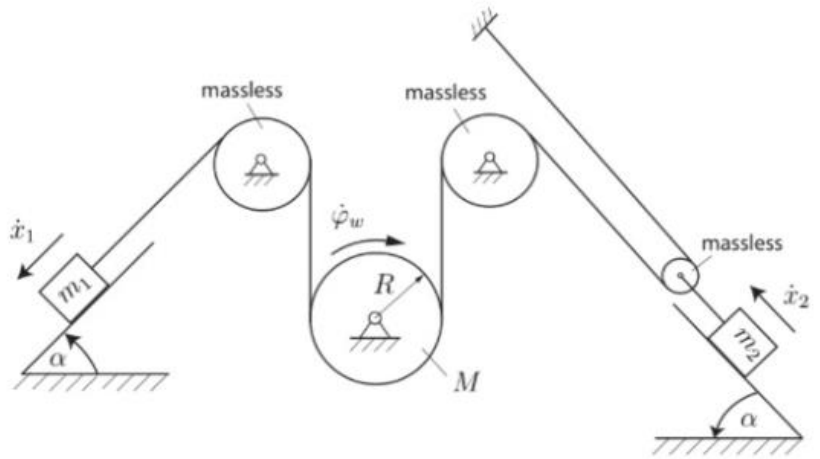
2 A crate of mass 300 kg is at rest on rough horizontal ground. The coefficient of friction between the crate and the ground is 0.5. A force of magnitude X N, acting at an angle α above the horizontal, is applied to the crate, where $\sin\alpha = 0.28$.

Find the greatest value of X for which the crate remains at rest.

Newton‘s Laws – Key Question

University Level

The figure below shows a pulley system consisting of two masses m_1 and m_2 which are sliding on inclined surfaces (inclination angle α), a homogeneous wheel (radius r, mass M) and three massless pulleys which are connected by a massless rope. The homogeneous wheel is fixed at its centre. Assume there is no slippage between the rope and the large wheel. The two masses can slide on the inclined planes without friction.



(a) Draw a free body diagram for each body.

(b) Write down the equations of motion for the masses m_1, m_2 and M using Newton's law.

(c) Write down the equation of motion for mass M using Euler's law.

(d) Write down a kinematic relation between mass m_1 and the homogeneous wheel, as well as for mass m_2 and the homogeneous wheel (on acceleration level).

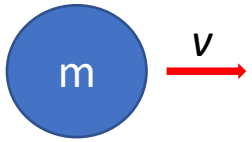
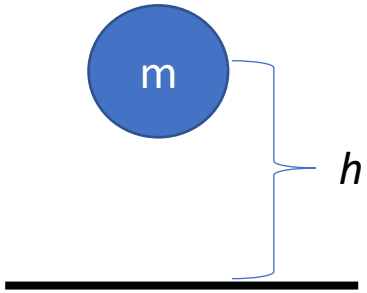
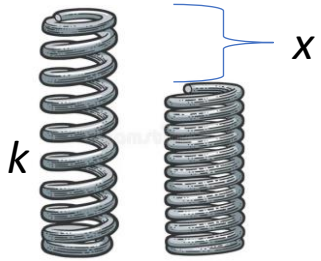
(e) Assuming that mass $m_1 = 10m$, $M = 4m$ and $m_2 = 8m$ determine the acceleration of the masses m_1,m_2,M and the tensions in the rope.

Energy, Work and Power

KEY POINTS – Must Remember

Conservation of Energy

$$Total\ Initial\ Energy = Total\ Final\ Energy$$

Kinetic Energy	Gravitational Potential Energy	Spring Energy
		
$KE = \frac{1}{2}mv^2$	$GPE = mgh$	$SE = \frac{1}{2}kx^2$

Energy, Work and Power – Key Question

High School Level

1 Two racing cars A and B are at rest alongside each other at a point O on a straight horizontal test track. The mass of A is 1200 kg. The engine of A produces a constant driving force of 4500 N. When A arrives at a point P its speed is $25\,\text{m s}^{-1}$. The distance OP is d m. The work done against the resistance force experienced by A between O and P is 75 000 J.

(a) Show that $d = 100$.

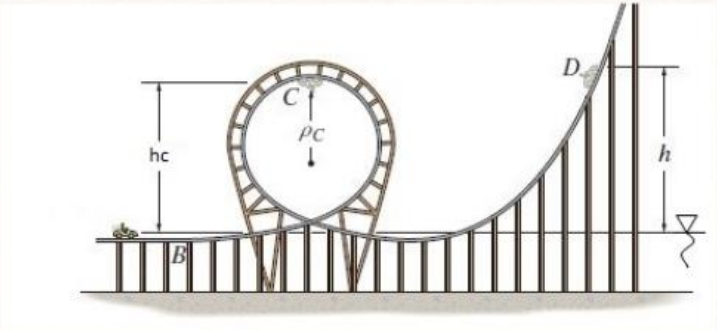
Car B starts off at the same instant as car A. The two cars arrive at P simultaneously and with the same speed. The engine of B produces a driving force of 3200 N and the car experiences a constant resistance to motion of 1200 N.

(b) Find the mass of B.

(c) Find the steady speed which B can maintain when its engine is working at the same rate as it is at P.

Energy, Work and Power – Key Question

University Level



(a) Determine the height h to the top of the incline D (with respect to the reference level given in the sketch) which the roller coaster car (idealized as point mass with mass m) will reach, if it is launched at B with a speed just sufficient for it to round the top of the loop C without leaving the track (falling down). The radius of curvature is ρ_C. The track is frictionless and there is no air resistance. The car is not firmly connected to the track. Use Conservation of energy!

(b) Determine the velocity at point B using conservation of energy.

Hints:

centripetal acceleration $a_{cp} = \omega^2 \cdot \rho_C$

Moment of inertia for a point mass $J = m \cdot \rho_C^2$

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